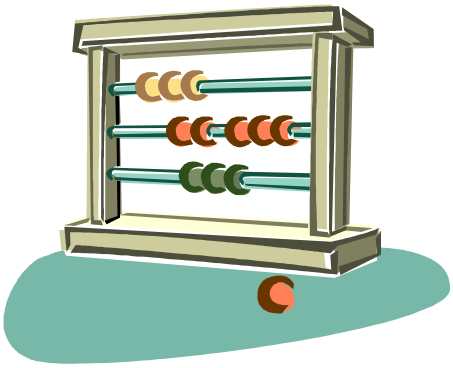
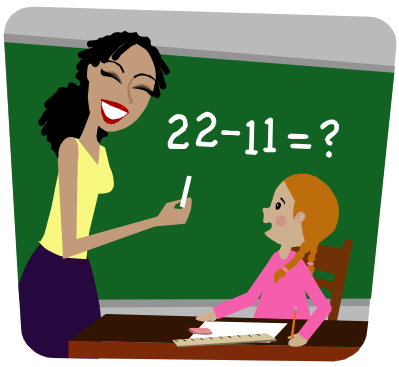
Mellor Maths

Progression in Calculation Strategies





In response to requests from parents we have put together the following information.

This booklet is designed to show you all the stages the children will be introduced to, as and when appropriate to their age and understanding, throughout Juniors (Key Stage 2). We hope this will give you an insight into the stages of their progression and enable you to make sense of what the children are doing.

There are key skills your child needs to know to help them improve their maths.

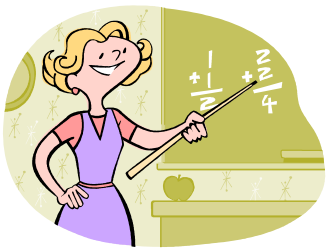
- Learning number facts by heart
- Knowing how to use what they already know to work out facts.
- Making jottings on paper so they don't lose track of what they're doing (it doesn't matter how rough or scribbly these are).
- Using images, or pictures, in their head to help them 'see' the maths more clearly. These images can include seeing numbers along a number line, up or down a number staircase, seeing patterns in numbers, imagining the units, ten and hundreds in different colours, or any other image that helps them.

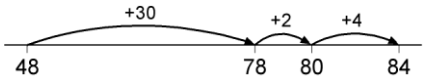
Below are some examples of the sorts of activities that you can practice with your child. Little and often is the best way to do it. Just 10 minutes every day of this kind of mental maths will help your child develop real mental maths fitness.

Learning facts by heart – what your child needs to know:

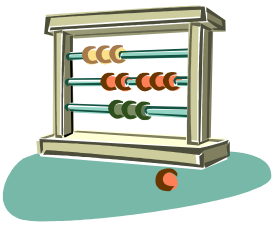
- **Number facts up to 10**
  Ask your child to tell you as many facts as they can in one minute, e.g. 4+6, 10-3, 5.7+4.3, 13.6-5.6 etc.
- **Number facts up to 20**
  Ask your child to tell you as many facts as they can in one minute, e.g. 8+7, 18-6 etc.
- **Pairs of numbers that total 100**
  Ask your child to tell you as many pairs as they can in half a minute, e.g. 75+25, 36+64 etc.
- **Pairs of numbers that total 1000**
  Ask your child to tell you as many pairs as they can in half a minute, e.g. 650+350, 200+800 etc.
- **Number facts up to 1**
  Ask your child to tell you as many facts as they can in half a minute, e.g. 0.2+0.8, 0.6+0.4 etc.
- **Doubling**
  Ask your child to double as many numbers as they can in a minute, e.g. double 36, double 3.8, double 480 etc.
- **Multiplying and dividing by 10, by 100, by 1000.**
  Give your child a number e.g. 30 and ask them to multiply it by 10, then 100, then 1000. Now give them a larger number e.g.6000 and ask them to divide by 10, then 100, then 1000.
- **Multiplication and division facts (times tables) up to 10 x 10.**
- **Square numbers**
  1x1=1; 2x2=4; 3x3=9, 4x4=16 etc. You can play a quick-fire challenge with these.

▪ Addition



Stage 1: The empty number line • **Mental methods lead to column addition.** • **Children need to be able to partition numbers in different ways.** • **The empty number line helps to record the steps.**	**8 + 7 = 15**  **48 + 36 = 84** 
Stage 2: Partitioning • **Add the tens and then the ones to find the total. This leads to column addition**	**Record steps in addition using partitioning:** **47 + 76** **40 + 7 + 70 + 6** **110 + 13 = 123** **Partitioned numbers can then be written under one another:** 47 = 40 + 7 +76 + 70 + 6 110 + 13 = 123
Stage 3: Expanded method in columns • **The expanded method leads children to the more compact method so that they understand it. The amount of time spent practising this method will depend on how confident children are in their understanding of place value.**	**Write the numbers in columns.** **Adding the ones first and then the tens:** 47 + 76 13 110 123
Stage 4: Compact column method • **In this method, carry digits are always recorded below the line, using the words 'carry ten' or 'carry one hundred', not 'carry one'.** • **Once learned, this method is quick and reliable.** • **This method would normally be covered by the end of Year 4.**	47 + 76 = 123 (11) 258 + 87 = 345 (11) 366 + 458 = 824 (11) **An aid to adding could be to circle the extra tens as each one is added in order to ensure they are not forgotten.**

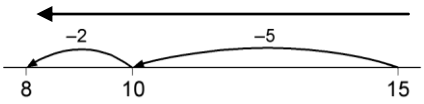
Subtraction



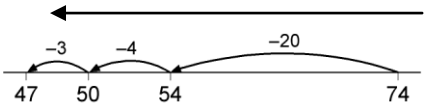
Stage 1: Using the empty number line

- **The empty number line helps to record or explain the steps in mental subtraction.**
- **Children will need to decide whether to count back or forward to *find the difference* and which is more efficient for calculations such as 57 – 12, 86 – 77 or 43 – 28.**

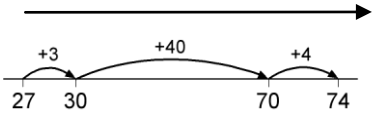
15 – 7 = 8



74 – 27 = 47 worked by counting back:



74 – 27 = 47 worked by counting forward;



Stage 2: Partitioning

- **Subtraction can be recorded using partitioning . Subtract the tens and then the ones to find the answer.**

- **Expanded method for 3 digit numbers.**

Record steps in subtraction using partitioning:

74 – 21 = 74 – 20 – 1
74 – 20 = 54
54 – 1 = 53

563 − 241

500 60 3
- 200 40 1
300 + 20 + 2 = 322

Stage 3: Decomposition

- **These steps lead children to the more compact method of decomposition. The amount of time spent practising the expanded method will depend on how confident the children are with partitioning.**

- **The movement of a ten into the units column is known as an *exchange.* (The ten is exchanged for 10 units). The language of "borrowing and paying back" is not used.**

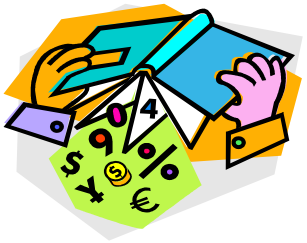
74 − 27

		60	14	6	14
70	4	~~70~~	~~4~~	~~7~~	~~4~~
- 20	7	-20	7	- 2	0
		40	7	4	7

563 − 271

500	60	3					
- 200	70	1					
400	160					4	16
~~500~~	~~60~~	3				~~5~~	~~6~~ 3
- 200	70	1				- 2	7 1
200	90	2				2	9 2
400	90	13					
~~400~~	~~100~~	3				4	9 13
~~500~~	0	3				~~5~~	0 ~~3~~
- 200	70	8				- 2	7 8
200	20	5				2	2 5

Multiplication



Stage 1: Multiplication using partitioning • **Mental methods for multiplying TU × U can be carried out using partitioning.** • **Either the tens or the ones can be multiplied first but it is more common to start with the tens.**	43 40 + 3 ↓ ↓ × 6 240 + 18 = 258 **or…** $14 \times 3 = (10 + 4) \times 3$ $= (10 \times 3) + (4 \times 3) = 30 + 12 = 42$ $43 \times 6 = (40 + 3) \times 6$ $= (40 \times 6) + (3 \times 6) = 240 + 18 = 258$
Stage 2: The grid method • **to multiply TU x U, first partition the TU number** • **Multiply T x U , then U x U and add together the products.** • **Place the number with the most digits in the left-hand column of the grid so that it is easier to add the products.**	38 x 7 × | 7 30 | 210 8 | 56 | 266
Stage 3: Expanded short multiplication • **This step records the calculation in columns showing the working.** • **Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is 'thirty multiplied by seven', not 'three times seven'.**	38 x 7 56 8 x 7 210 30 x 7 266
Stage 4: Short multiplication • **The recording is reduced further, with carry digits below the line.**	38 × 7 266 5

Stage 5a: Grid method

- **This method can also be used for**
  TU x TU and HTU x HTU

56 x 27

x	20	7
50	1000	350
6	120	42
	1120	392

1120
+ 392
1512

Stage 5b: Expanded method

- **This expanded method can also be used for TU xTU**

56
x 27
42 6 x 7
350 50 x 7
120 6 x 20
1000 50 x 20
1512

Stage 6: Long multiplication

- **TU x TU should be set out like this**

56
x 27
392 56 x 7
1120 56 x 20
1512
1

Stage 6 will be introduced in exceptional cases for Gifted and Talented pupils in Year 6.

286
x 29
2574
5720
8294

Division

Division can be thought of as sharing or grouping. It is important that both concepts are explored with the children in order that they gain a full understanding of the operation.

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Stage 1: Mental division using grouping
• Mental methods for dividing TU ÷ U can be based on children's recall of multiplication and division facts
• When dividing a larger number , children should use the facts they know to group the number,
Ie For 84 ÷ 7=? I know 10 lots of 7 is 70, and there are 14 left over, so 10 lots of 7 and 2 lots of 7 makes 12.
• Children should also be able to find a remainder mentally, for example the remainder when 34 is divided by 6.
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84 ÷ 7 =

84
70 + 14
↓ ↓ ÷ 7
10 + 2 = 12

34 ÷ 6 = 5 R 4
6 x 5 = 30
34 – 30 = 4
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Stage 2: Expanded method for TU ÷ U / HTU÷U or "chunking"
In 97 ÷ 9 we refer to 97 as the dividend and 9 as the divisor
• In this method children take 'chunks' out of the dividend. The 'chunks' are multiples of the divisor, usually 10 lots at a time.
• Once they understand 'chunking' children should be able to move on from TU ÷ U to HTU ÷ U quite quickly

• As children progress, encourage them to reduce the number of steps and move them on quickly to finding the largest possible multiples.
• The key to the efficiency of chunking lies in the estimate that is made before the chunking starts.

This would normally be covered by the end of Year 4.
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97 ÷ 9

9⟌97
– 90 (9 x 10)
7
Answer: 10 R 7

6⟌196
-60 (6 x 10)
136
- 60 (6 x 10)
76
- 60 (6 x 10)
16
- 12 (6 x 2)
4 32
Answer: 32R4

6⟌196
- 180 (6 x 30)
16
- 12 (6 x 2)
4 32
Answer: 32R4
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Stage 3: Short division of TU ÷ U • 'Short' division of TU ÷ U is a more compact recording. • Children need to understand that in this layout the answer is written above the line. • Short division of a two-digit number is introduced to children who are confident with place value, multiplication and division facts and subtracting multiples of 10 mentally	$\begin{array}{r} 2\ 7 \\ 3\overline{)8^{2}1} \end{array}$ **The carry digit '2' represents 2 tens.**
Stage 4: Short division of HTU÷ U	$\begin{array}{r} 9\ 7 \\ 3\overline{)2\ 9^{2}1} \end{array}$
Stage 5: Chunking Method for HTU÷TU **The next step is to tackle HTU ÷ TU** **This is done in the same way as above, but with larger numbers.** **This example explains how children might use this method to solve a problem.**	**How many packs of 24 can we make from 560 biscuits?** **Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs.** **Use this to start by subtracting 480 from 560.** $\begin{array}{rll} 24\overline{)\ 560} & & \\ -\ \underline{480} & (24 \times 20) & \\ 80 & & \\ \underline{72} & (24 \underline{\times\ 3}) & \\ 8 & & 23 \end{array}$ Answer: 23R8 **Once the answer is found, interpret it in context of the problem, so the answer is 23 packs of biscuits.**